### IMPERIAL COLLEGE LONDON

## COMPUTER LAB ASSIGNMENT REPORT

## Building, Simplifying and Evaluating Binary Trees

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### Part I

## The struct array\_aux

### 1 General Introduction

In the assignment, the students were asked to provide a piece of C++ coding, that would build a tree with the minimum number of nodes, and would evaluate the built trees. Simplification must take place to reduce the number of nodes in the tree. Two common approaches are to build first and simplify after, or to simplify first and build after.

I will be using the latter case, which is simplifying first and then building the tree.

The number of characters n in each string input is passed into the constructor of array\_aux. The user inputs are stored in the seq\_2D\_char\_input matrix, implemented using a 2D char array. The simplification The 1D int array, will then take place.  $seq_{-}1D_{-}int$ , is of length  $2^{n}$ . In the 1D int array, the elements hold the default value of -1. If an input, for example, 1001, is passed in, the program will first convert 1001 to 9. The binary to decimal conversion is done outside the struct, using a function called binaryToInt. The program will go to the 9th location in the seq\_1D\_int array and set the value at that location to a number and the number is the index in the seq\_2D\_char\_input matrix where the input is stored. Every time an input is stored, the program will find the next location in the matrix where no valid records are stored. The index of that location is stored in the seq\_1D\_int array at the location that is equal to the decimal value of the input binary string. In

other words, the int array is used as a mapping, to show the location of the record in the matrix. It is a way to reduce the size of the matrix. Suppose if the matrix is initialized with a dimension of  $2^n * n$ , while there are only 5 inputs. In this case,  $2^n - 5$ rows are wasted, which is not RAM efficient. Hence the program uses a list of integers to show the index of the record in the record matrix. The size of the record matrix, seq\_2D\_char\_input, can be reduced to the total number of inputs from the input vector. Also, seq\_1D\_2D\_available\_space is used to show whether there is a record at the index given in the seq\_2D\_char\_input matrix, so that the program does not have to compare every element in a record in the matrix to determine whether the record is valid or not.

By hiding the complexity of storing the input binary strings, the data structure can be viewed in table 2. When the program wants to access a record whose decimal value is k, the program will first go to the kth location in the int array and retrieve the information stored at that location. If the retrieved number j is not -1, meaning it holds a valid record, the program will then go to the *jth* location in the char matrix to get the exact information in the record. If a record is to be deleted, the jth element in the  $seq_1D_2D_available_space$  is set to false. In the following explanation of the program, the way of describing data structure is in table 2. The actual implementation is different from implementing a matrix like table 2.

Name	Type	Dimension	Explanation
$input\_string\_length$	int		number of characters in each input string
$array\_length\_1D$	$\operatorname{int}$		total number of different possible inputs
$vector\_len$	$\operatorname{int}$		number of elements in the input vector
$seq\_1D\_int$	int array	$2^n$	array stores the indices of routes
$seq\_2D\_char\_input$	char matrix	$vector\_len*n$	matrix stores the input
$seq\_1D\_2D\_available\_space$	bool array	$vector\_len$	shows if a record exists in the 2D char table
$seq_2D_int_sort$	int matrix	n*6	used in sorting

Table 1: Members of the struct array\_aux

	bit 0	bit 1	bit 2	bit 3	 bit n-1
row 0	$a_{0,0}$	$a_{0,1}$	$a_{0,2}$	$a_{0,3}$	$a_{0,n-1}$
row 1	$a_{1,0}$	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$	 $a_{1,n-1}$
row 2	$a_{2,0}$	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$	 $a_{2,n-1}$
row 3	$a_{3,0}$	$a_{3,1}$	$a_{3,2}$	$a_{3,3}$	 $a_{3,n-1}$

Table 2: Data structure that stores the inputs with dimension  $2^n * n$ 

	bit 0	bit 1	bit 2	bit 3
record 9	1	0	0	1

Table 3: Example: input = 1001 (9 in decimal), stored in the seq\_2D\_char\_input matrix

In the  $seq_2D_char_input$  matrix, if a record exists, the corresponding bits will be set according to the binary strings input by the user. The matrix contains all the possible combinations of different bits. Each bit in the matrix will take one of the 3 possible values, 0, 1 or 2. To set the bits in the matrix, the binary string is converted to the corresponding decimal value, denoted as k. Then the program will go to the kth row in matrix. The row has the identical length as the length of each individual input string. Each bit in the row is set to either 1 or 0 based on the binary. Also, in the seg\_1D\_2D\_available\_space array, the element that links to the record is set to true.

The input binary strings are stored in

a vector. The program will iterate through all the strings in the vector. For the rows in the seq\_2D\_char\_input matrix that are not operated on, the default value of the bits in those rows is 2. During simplification, the bits that originally hold a 1 or 0 can be set to 2 as well. If a row is filled with 2, the row does not hold a record and does contribute to the structure of the simplified binary The corresponding elements in the seq\_1D\_2D\_available\_space array will have values of false. To check whether a record exists at a given index m, the program does not have to check all the bits in the mth row in the matrix. Instead, using the value at the mth element in the  $seq_{-}1D_{-}int$  array as an index and using the index to check in

the  $seq_1D_2D_available\_space$  array is sufficient.

The  $seq_2D_int_sort$  matrix has a different dimension from the previously explained  $seq_2D_char_input$  matrix. The  $seq_2D_int_sort$  matrix holds information on certain bit position in any binary string, instead of each binary string from the input.

A row number q in the matrix corresponds to the qth bit in any input string. The values in each row in the matrix are expianed in table 5. Note that the values in the  $seq\_2D\_int\_sort$  matrix are set after simplification. Bit 2 and bit 3 in row q are set by counting the number of 1s and 0s in the  $seq\_2D\_char\_input$  matrix at column q.

	bit 0	bit 1	bit 2	bit 3	bit 4	bit 5
row 0	$a_{0,0}$	$a_{0,1}$	$a_{0,2}$	$a_{0,3}$	$a_{0,4}$	$a_{0,5}$
row 1	$a_{1,0}$	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$	$a_{1,4}$	$a_{1,5}$
row 2	$a_{2,0}$	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$	$a_{2,4}$	$a_{2,5}$
row 3	$a_{3,0}$	$a_{3,1}$	$a_{3,2}$	$a_{3,3}$	$a_{3,4}$	$a_{3,5}$
row n	$a_{n,0}$	$a_{n,1}$	$a_{n,2}$	$a_{n,3}$	$a_{n,4}$	$a_{n,5}$

Table 4: Structure of the  $seq_2D_int_sort$  matrix with dimension n\*6

	bit 0	bit 1	bit 2	bit 3	bit 4	bit 5
row n	$\mid n \mid$		counter: bit $n = 1$	$\begin{array}{c} \max \\ \text{of bit } 1\&2 \\ \text{at row } n \end{array}$		weight of the bit $n$

Table 5: Explanation of each bit in the seq\_2D\_int\_sort matrix

Function	Description
array_aux(int len, int len_of_vector)	constructor, initialize the members in the struct
$\sim$ array_aux()	destructor, release memory
int return_array_length_1D()	return the member array_length_1D
void seq_1D_int_set(int position, bool val)	change the element in the seq_1D_int array
int seq_1D_int_return(int position)	return one element in the seq_1D_int array
$void seq_1D_int_test()$	print the elements with index in the seq_1D_bool array
void seq_2D_char_input_set (int position, std::string binary_string)	set one row in the seq_2D_char_input matrix at given index using given string
void seq_2D_char_input_set_bit (int position, int bit, char info)	set certain bit in the seq_2D_char_input matrix using the given index
char seq_2D_char_input_return(int x, int y)	return the element at give index in the seq_2D_char_input matrix
int seq_2D_char_input_weight(int position)	treat 2 as 1 and calculate the decimal value at give index
void seq_2D_char_input_test()	print the seq_2D_char_input matrix
int check_available_routes()	check the number of routes that will be used in building the tree
void delete_route(int position)	delete a route at give index
void seq_2D_int_sort_bit_set()	update elements in the seq_2D_int_sort matrix using the seq_2D_char_input matrix
void seq_2D_int_sort_bit_sort(int bit_sort)	sort the seq_2D_int_sort matrix using a certain column
void seq_2D_int_sort_bit_run()	sort the seq_2D_int_sort matrix using column 4 first, followed by column 3
int seq_2D_int_sort_return(int x, int y)	return one element in the seq_2D_int_sort matrix
void seq_2D_int_sort_test()	print the seq_2D_int_sort matrix
void seq_2D_int_sort_priority_gen()	generate the priority of each bit and store them in the seq_2D_int_sort matrix

Table 6: Member functions in the struct  $array\_aux$ 

### 2 The Constructor

The function  $array_aux()$  is the constructor of the struct array\_aux and takes in two arguments, which is the length of one individual input binary string (denoted as h), and the size of the input vector (denoted as v). The constructor first set the input\_string\_length member to h and  $vector\_len$  to v. Then the constructor calculates array\_length\_1D, which is the total number of possible combinations, using the formula  $2^h$ .  $array\_length\_1D$ and input\_string\_length are used to initialize the  $seq_1D_2D_available_space$  array, the seq\_2D\_char\_input matrix and the seq\_2D\_int\_sort matrix. The dimensions of those arrays and matrices can be found in table 1.

### 3 Check Available Routes

The function check\_available\_routes() is used to count how many elements that hold the value of true in the seq\_1D\_bool array. The returned result indicates the number of available routes that can be used to build the simplified the binary tree. The returned value will change when the function is called at different simplification cycles. Initially, the function is invoked before and after a simplification cycle. If the returned values are the same, then the whole simplification process has reached an end and no more simplification is possible. The program will then move onto the building stage.

### 4 Delete Routes

The function delete\_route(int position) is used to delete a route at a given in-

dex. To delete the route, the function will set all the bits in the respective row in the  $seq_2D_-char_-input$  matrix to 2 and the respective element in the  $seq_1D_2D_-available\_space$  array to false.

## 5 Collect, Sort, Generate

The function  $seq_2D_int_sort_bit_set()$ will count the number of 1s and 0s at certain bits. In other words, the function counts the number of 1s at bit 0, bit 1, bit 2...until the last possible bit among all the available routes. And store the result in the third element at row 0, row 1, row 2...until the last row in the seq\_2D\_int\_sort matrix. Then the program will do the same for counting 0s, but the result is stored in the second element in each row in the matrix. The fourth element in each row will store the bigger number by comparing the second and the third element in the same row. The fifth element will be the sum of the second and the third element in the same row. The sixth element in the row is the priority of the bit position associated with the row. The first element in the row of the matrix is the row number (counting from 0) plus 1, since the node numbering in the final built tree starts from 1, instead of 0.

The function  $seq\_2D\_int\_sort\_bit\_sort(int\ bit\_sort)$  will arrange the rows in the  $seq\_2D\_int\_sort$  matrix. The principle of rearranging is comparing elements in a column of the mateix. The column number will be given when invoking the function. The algorithm used is bubble sort.

The function  $seq_2D_int_sort_bit_run()$ will run the function  $seq_2D_int_sort_bit_sort(int bit_sort)$  twice, but the parameters passed in are 4 first followed by 3. This means that the rows in the seq\_2D\_int\_sort matrix will be sorted twice, first by the column showing the sum, followd by the column showing the maximum.

seq\_2D\_int\_sort\_priority\_gen() is used to generate the priority of each bit position and fill in the sixth column in the seq\_2D\_int\_sort matrix. The priority starts from 1 and increment by 1 every time a new and lower priority level is reached. The smaller the value of the priority level is, the higher the priority will be. Two bit positions, or two rows,

will only have the same level of priority if and only if the max and the sum elements are the same. For instance, the max of row 4 equals that of row 5, and the sum of row 5 will have the same priority. If any of the two requirements are not satisfied, the priority generated will be different. Rows on the top of the matrix will have the highest priority.

Table 7 to Table 11 show the way of rearranging and priority generating.

	bit 0	bit 1	bit 2
row 0	2	2	2
row 1	0	0	1
row 2	2	2	2
row 3	2	2	2
row 4	1	0	0
row 5	2	2	2
row 6	2	2	0
row 7	1	1	1

Table 7: Demo: input table

	bit 0	bit 1	bit 2	bit 3	bit 4	bit 5
row 0	0	1	2	2	3	0
row 1	1	2	1	2	3	0
row 0 row 1 row 2	2	2	2	2	4	0

Table 8: Demo: seq\_2D\_int\_sort matrix, count for 0s and 1s updated

	bit 0	bit 1	bit 2	bit 3	bit 4	bit 5
row 2	2	2	2	2	4	0
row 2 row 0 row 1	0	1	2	2	3	0
row 1	1	2	1	2	3	0

Table 10: Demo: seq\_2D\_int\_sort matrix, sort using the max (bit 3)

	bit 0	bit 1	bit 2	bit 3	bit 4	bit 5
row 2	2	2	2	2	4	0
row 0	0	1	2	2	3	0
row 2 row 0 row 1	1	2	1	2	3	0

Table 9: Demo: seq\_2D\_int\_sort matrix, sort using the sum (bit 4)

	bit 0	bit 1	bit 2	bit 3	bit 4	bit 5
row 2 row 0 row 1	2	2	2	2	4	1
row 0	0	1	2	2	3	2
row 1	1	2	1	2	3	2

Table 11: Demo: seq\_2D\_int\_sort matrix, priority added

#### Part II

## buildcompactbdt

## 1 Loading Inputs

In the first stage of building the tree, an  $array\_aux$  object is instantiated and is called  $array\_gen$ . The parameter passed in is the length of one individual input string. To load the binary strings, a For loop is used to iterate through the elements in the input string vector. Function binaryToInt is invoked to convert the binary string to a decimal integer. The input string will then be stored in  $array\_gen$  at index that corresponds to its decimal value.

#### 2 Extreme Cases

Two extreme cases exist after all the inputs are loaded. The first one is that the number of valid records is 0, meaning regardless of the inputs, the tree will always produce a 0 at the output. The solution to this extreme case is to build a tree with only one node that has the value of 0. The pointer of the node is returned and the build-compactbdt() function is terminated.

The other extreme case is that the number of valid records equals the total number of possible inputs. In other words, the binary tree will always produce a 1 regardless of the inputs. The solution is to build a tree with only one node that has the value of 1. The pointer of the node is returned and the buildcompactbdt() function is terminated.

# 3 Calculating Weights and Simplification

After loading inputs comes the simplifying stage. The principle is simple Boolean algebra. If two records differ by only 1 bit while both of them give out a 1 as the output. Then the different bit can be treated as the don't care bit and can be removed in both records, resulting in a route that is 1 bit shorter than the original route. The second record in the comparison is deleted by calling the member function delete\_route using the index of the second record.

Question on finding the index of the second record arises. Since the two records only differ by one bit. And the bit position is known. Hence technically the index of the second record can be calculated. If the second record does not exist, or it does not produce a output as 1, then the comparison does not produce further simplification at certain bit position with certain record. The program will move on to the next record that will show a 1 at the output.

Before simplification:

	bit 0	bit 1	bit 2
row 0	2	2	2
row 1	2	2	2
row 2	0	1	0
row 3	2	2	2
row 4	2	2	2
row 5	2	2	2
row 6	1	1	0
row 7	2	2	2

After simplification:

	bit 0	bit 1	bit 2
row 0	2	2	2
row 1	2	2	2
row 2	2	1	0
row 3	2	2	2
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

In the above example, row 2 and row 6 are being compared. The pointer for the records traverses from the top to the bottom of the matrix. In other words, the record pointer moves from smaller index to larger index in each cycle of simplification. And horizontal pointer for the bit moves from the left to the right. In this demonstration, the record pointer points to row 2 and the bit pointer points at bit 0. The second record for comparison should locate at distance of  $2^2 = 4$  after the first record. The first record for comparison locates at index 2. Hence the second record for comparison locates at index 6. At index 6, a record exists that will produce a 1 at the output. Hence, for the first record, or the record at index 2 in the matrix, its bit 0 can be treated as don't care case and can be removed from the record at index 2. The record at index 6 is removed as well as a result of the simplification.

Each simplification operation can only simplify one bit in one record at one time. Once the record is modified, the program will move on to the next available record. Multiple operation on the same record is not allowed in the same simplification cycle. The program will start over, or start a new simplification cycle, from the very first record in the input matrix once it has reached the end of the matrix. To determine

whether the table has reached its most simplified form, the program will do an available route counting before and after each simplification cycle. If the counts possess the same value, then the whole simplification process is done and the program will move to the stage of sorting the routes.

During simplification, some routes will be deleted and some nodes in the route will be set to 2. The bit labeled as 2 does not contribute to building the simplified tree. In the program, a 2 in a simplified route is treated as 1. Hence the weight, or the corresponding decimal value of that route does not equal to the index of the route. See an example below for explanation.

	bit 0	bit 1	bit 2
row 0	0	0	0
row 1	0	0	1
row 2	0	1	0
row 3	0	1	1
row 4	1	0	0
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

The number of valid Each cycle of simplification starts from row 0. The number of valid records is 5. At row 0 bit 0, a 0 is present. The weight of row 0 now is 0. The distance is  $2^2 = 4$ . The row for comparison locate at row 0+4=4. Row 4 has a weight of 4, which is the same as the weight of row 1 plus the distance.

	bit 0	bit 1	bit 2
row 0	2	0	0
row 1	0	0	1
row 2	0	1	0
row 3	0	1	1
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

Simplification between row 0 and row 4 takes place. All elements in row 4 is deleted by setting them to 2. Also, at row 0 bit 0, it is set to 2 to indicate the bit is removed from the route.

	bit 0	bit 1	bit 2
row 0	2	0	0
row 1	0	0	1
row 2	0	1	0
row 3	0	1	1
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

Each row can only have one simplification in each simplification cycle. Hence the program will move to row 1. The weight of row 1 now is 1. The program now looks at row 1 bit 0, the distance is  $2^2 = 4$ . Hence the row for comparison is row 1 + 4 = 5. However, row 5 is not a valid record since all its elements are 2.

	bit 0	bit 1	bit 2
row 0	2	0	0
row 1	0	0	1
row 2	0	1	0
row 3	0	1	1
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

The program now looks at row 1 bit 1.

The corresponding distance is  $2^1 = 2$ . Hence the row for comparison is row 1+2=3. The weight of row 1 now is 1. The weight of row 3 now is 3, which is the same as the weight of row 1 plus the distance.

	bit 0	bit 1	bit 2
row 0	2	0	0
row 1	0	2	1
row 2	0	1	0
row 3	2	2	2
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

Simplification takes place between row 1 and row 3. Row 1 bit 1 is set to 2 and row 3 is deleted by setting all elements to 2.

	bit 0	bit 1	bit 2
row 0	2	0	0
${\rm row}\ 1$	0	2	1
row 2	0	1	0
row 3	2	2	2
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

The program now moves to row 2. Row 2 bit 0 is 0 and the distance calculated is 4. Hence the row for comparison is row 6. However, row 6 is not a valid route. The program then moves to row 2 bit 1. But the matrix produces a 1 at this location. Hence this location is skipped for simplification.

	bit 0	bit 1	bit 2
row 0	2	0	0
row 1	0	2	1
row 2	0	1	0
row 3	2	2	2
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

The next bit for comparison is row 2 bit

2. The distance calculated is  $2^0 = 1$ . Hence the row for comparison is row 3. But row 3 is not a valid record. This cycle of simplification reaches an end since row 2 is the last row that produces a valid record. The number of valid records is 3, which is difference from the number of valid records at the start of this simplification cycle. Hence another cycle will start from the first valid record, which is row 0 in this case.

4, 1111011 10 1011 0 111 01110 0000			
	bit 0	bit 1	bit 2
row 0	2	0	0
row 1	0	2	1
row 2	0	1	0
row 3	2	2	2
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

At the start of this simplification cycle, the number of valid records is 3. Since 2 is treated as 1 in this algorithm, the first position for comparison is row 0 bit 1. The corresponding distance is 2. The weight of row 0 now is 4. The row for comparison is row 2

with weight 2, which is not the same as the weight of row 0 plus the distance. Hence the simplification does not take place between row 0 and row 2 at bit position 1.

	bit 0	bit 1	bit 2
row 0	2	0	0
row 1	0	2	1
row 2	0	1	0
row 3	2	2	2
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

The next comparison is between row 0 and row 1 at bit 2. The weight of row 0 is 4. The distance is 1. The weight of row 1 is 3, which is not the same as the weight of row 0 plus the distance. Hence no simplification takes place.

The remaining discussion on whether the simplification will take place follows the same pattern. In this case, no other simplification cycle is possible. The final matrix will be the following:

	bit 0	bit 1	bit 2
row 0	2	0	0
row 1	0	2	1
row 2	0	1	0
row 3	2	2	2
row 4	2	2	2
row 5	2	2	2
row 6	2	2	2
row 7	2	2	2

Row 0, 1 and 2 will be used for building the binary tree.

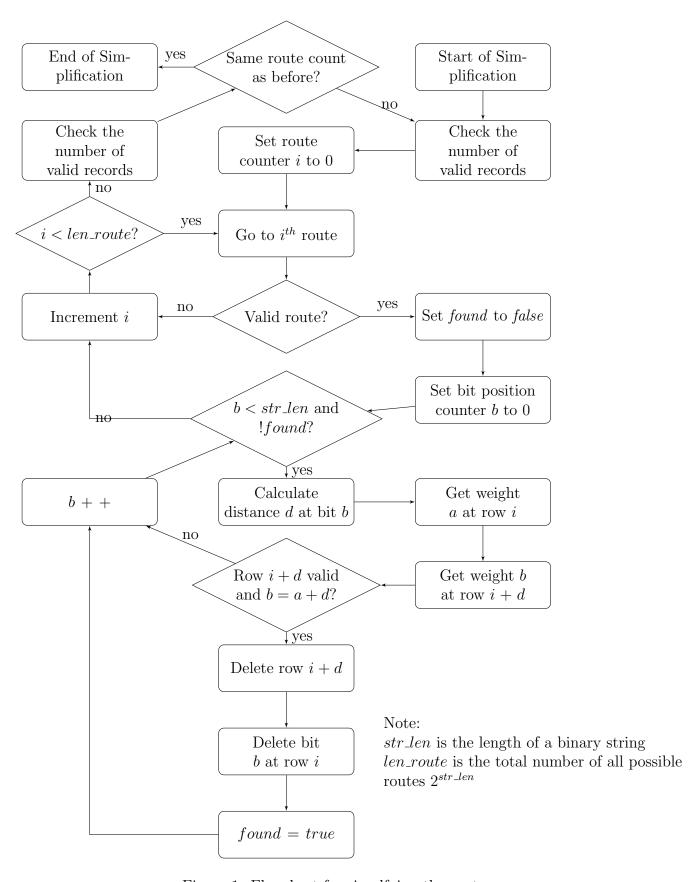


Figure 1: Flowchart for simplfying the routes

## 4 Sorting Routes

Member functions  $seq_2D_int_sort_bit_run()$  and  $seq_2D_int_sort_priority_gen()$  will be invoked for generating the priority of each bit position.

The program will then initialize a t \*t is the number of availe-2 matrix. ble routes, which can be obtained using the check\_available\_routes() member function. In each row of the matrix, the first element will be the route/valid record position and the second element will be the calculated weight of the route. The matrix does not hold the specific information on the route but the priority level and the index of the route. The priority is calculated using the priority generated for each bit position. Note than a 2 in the route record does not contribute to the weight of the route but a 1 or a 0 will contribute to the weight. In other words, when there is a 0 or a 1 in a certain bit position in a certain record, the weight or the priority of the bit position is added to the total weight of the route. Once the matrix that holds the route index and the route weights is generated, it will be sorted with bubble sort using the weight of each route as the sorting key.

In program's implementation, a low weight, or a small value in the priority indicates a high priority, meaning the route associated with the weight should be built prior to the remaining routes. The first route with the highest priority will be built first so that the other routes can have a place to build upon.

## 5 Building Routes and Filling 0s

Since there are nodes in a route that will be ignored, each route could have different lengths. In order to determining the end of a route, the number of nodes that will be used in building has to be available before building the route. The number of useful nodes in a route is determined by checking all the emelents in the route using a For loop. The counter is set to be equal to the length of a binary string (the length of each string from inputs). If the element is a 2, the counter will be decremented. Back to building the tree, every time a node is added to the route, the counter for the no-ignore nodes will decrement. The route will reach an end when the counter reaches 0 and in the end of the route, a 1 will be added to the label of the last non-leaf node. The sequence of the nodes in building each route, from the top to the bottom of the tree, will be following the node, or the bit priorities, which is determined previously by calling the member functions. The standard procedure of building a route will be getting the route index, getting the bit position in the bit priority sequence, and checking the corresponding bit in the route. If the result is a 1, then a new node will be generated and labeled with the bit position, by calling the node-Namer() function. Then the pointer will point to the right of the current node. If the result is 0, then the same procedure will apply other than the pointer will be pointing to the left of the current node. If the result is 2, then no new node will be added to the tree. The pointer will stay at the current position.

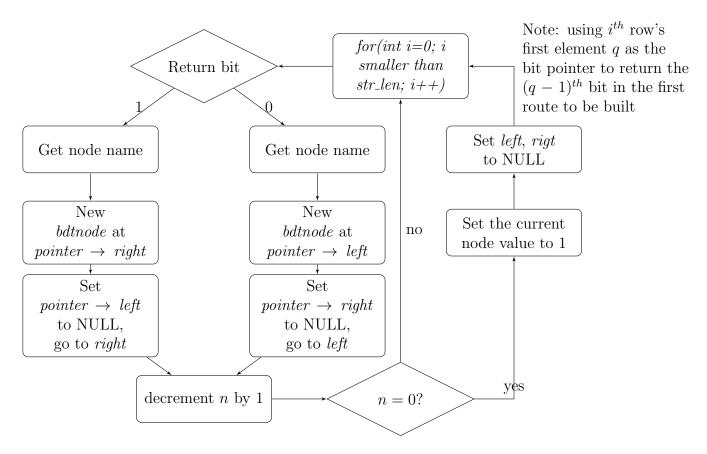


Figure 2: Flowchart of building the first route

After the first route is done, the program will move onto building other available routes. The difference between building the first route, and those that come after the first one, is the node name check. The node name, or the bit associated with the node name, will be used to guide the building process. If the bit position in the route is 1, then the pointer will point to the right of the current node and move on. If it is a 0, then the pointer will point to the left and move on. If it is a 2, the program will treat it as a 1,

meaning the pointer will point to the right of the current node. A 2 indicates a don't care case and it does not matter which direction it will lead the pointer. However, the direction has to be consistent. It is not allowed that a 2 will lead the pointer to either the left or the right in the same tree.

In the building process, only 1s are filled at the leaf nodes. The missing 0s at the leaf nodes will be filled by calling the function nodeSetZero(). The root of the binary tree is returned as the output.

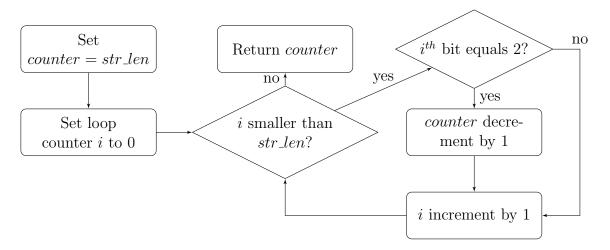


Figure 3: Flowchart of counting the number of nodes that contributes to building the tree

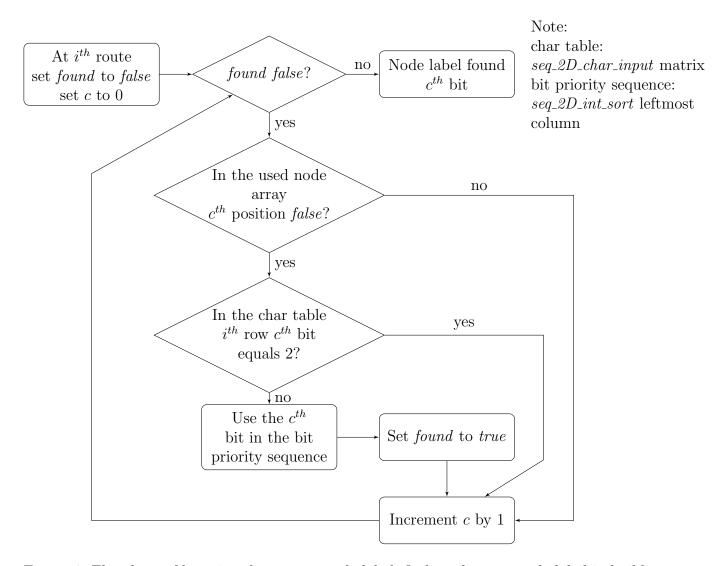


Figure 4: Flowchart of knowing the current node label, finding the next node label in building the route

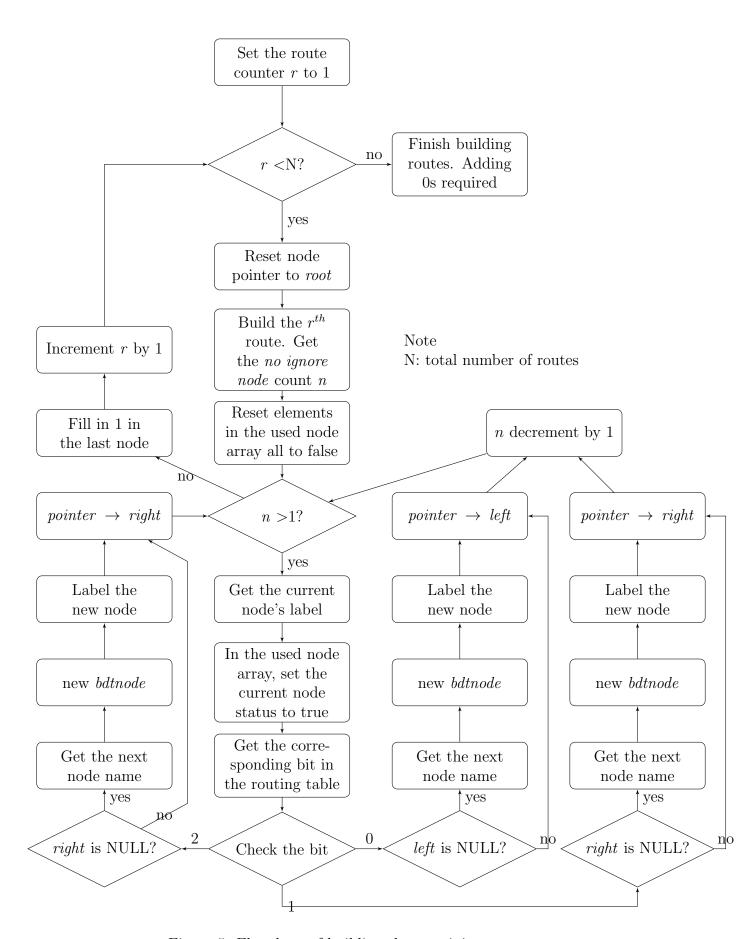


Figure 5: Flowchart of building the remaining routes

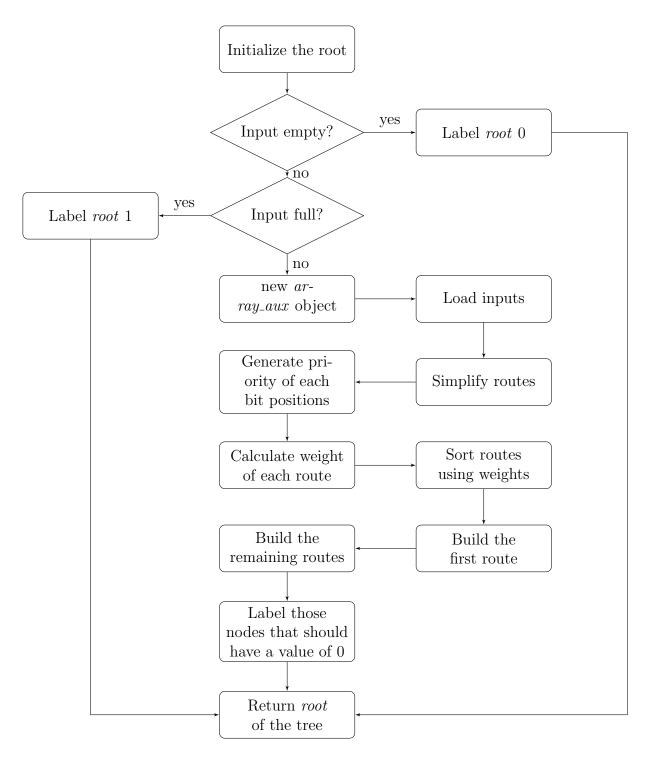
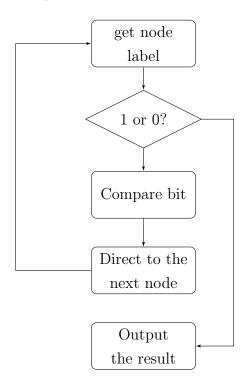


Figure 6: Flowchart of the function buildcompactbdt

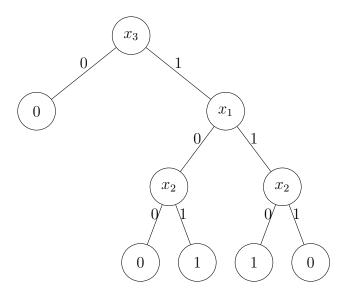
## Part III

## evalcompactbdt

The function evalcompactbdt will start from the root of the tree. In each iteration before getting a 1 or a 0, the function will get the number in the node name after the character x, using the substr method in the string library. There is a difference of 1 between the integer returned from trimming the node name and the bit it points to. Hence the returned integer number has to be deducted by 1. The integer will then be used as a pointer to find the corresponding value in the input string. If at that bit it is a 1, then move to the right. Else if it is a 0, then move to the left. The iteration is controlled using a while loop. When a 1 or a 0 is reached, the boolean variable that controls the while loop will be set to true and the loop will be terminated. The loop will be running if the boolean variable is false. The final value at the node will be returned as the output.



During the building process of the tree, the nodes are not always arranged in order. In other words, the nodes on the top does not always start from x1. The order is random while the rule of finding the output is fixed. The label of each node can lead to a bit in the input string. Hence a simple look up and check algorithm is implemented here.



An example here will be to use the binary string 101 and the above tree for evaluation. The label on the root of the tree is x3. The third bit, counting from the left, the character is 1. 1 indicates going to the right of the tree. The next node is labeled as x1. The first bit in the input string is 1. Hence the next step is to move to the right of the node. The next node is x2. Checking the second bit in the string outputs 0. Hence move to the left of the x2 node. A final value is shown here, which is 1. Therefore, using the binary string 101 and the above tree yield an output of 1.

### Part IV

## Other Functions

Function	Description
int binaryToInt(const std::string num)	convert a binary to an decimal integer
std::string nodeNamer(const int level_label)	output a string with x appended before the integer
void nodeList(bdt t)	list all the nodes in the tree
void nodeSetZero(bdt t)	set the 0 values in the simplified tree
int nodeCounter(bdt t)	count the number of leaf nodes
void nodeCountAux(bdt t, int& count)	auxiliary function in counting nodes that traverses the tree recursively

Table 12: Other functions outside the struct array\_aux

## 1 binaryToInt

The algorithm is to calculate the weight of each bit in the binary string and sum the weights up to give the output. The function will iterate through all the bits in the binary string, starting from the right most bit. An int variable is initialized to be 0 and used to hold the result as the iteration continues.

	bit n	 bit 2	bit 1	bit 0
weight	$2^n$	 $2^2 = 4$	$2^1 = 2$	$2^0 = 1$

The calculated result will be to use the bit at the corresponding position to multiply the weight at that bit position. And then add up all the weights to obtain the final result.

An example here will be to calculate the decimal value of the binary string 1011.

	bit 3	bit 2	bit 1	bit 0
weight	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

The result will be 1\*8+0\*4+1\*2+1\*1 = 11.

## 2 nodeNamer

The function is to add an x in front of the input integer and output the string. It initializes a stringstream variable and inputs the x followed by the integer. The stringstream variable is returned after calling the str() method.

## 3 nodeList

The function is recursive. Its main body is an If statement to determine whether the passed in pointer points to a NULL position or not. If the pointer does not points to a NULL position, the next statement will be to call itself using the pointer pointing to the right of the current node. Then a print statement is used to show the label on the current node. The last statement in the If statement block is to call the *nodeList* function using the pointer pointing to the left of the current node. The function is used to display all the nodes in the binary tree, from the rightmost to the leftmost.

#### 4 nodeSetZero

The function is used in building the tree. As discussed preciously, the buildcompactbdt function will only fill in the 1s at the end of building each route. A lot of positions in the binary tree that remain as NULL while actually should be 0. The function nodeSetZero is used to change those locations to 0. The locations that should be labeled as 0 should possess the following properties. Its parent node's label starts with x. In other words, its parent is a non-leaf node in the tree. The information on the node is NULL. (Refer to Figure 1)

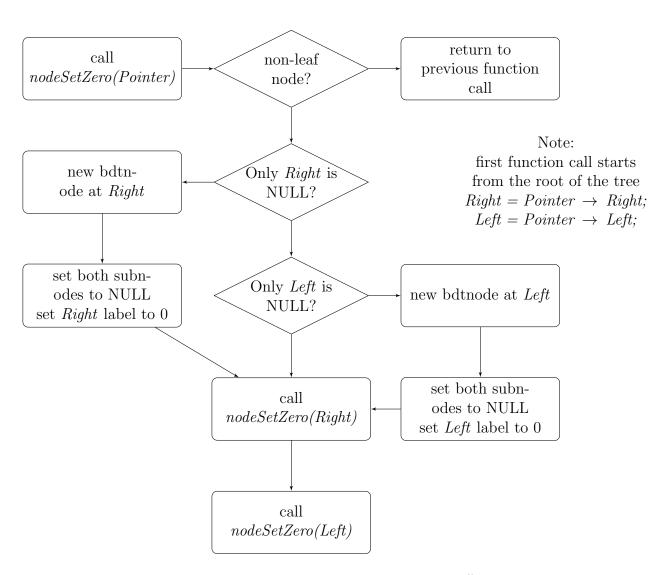


Figure 7: Flowchart for nodeSetZero()

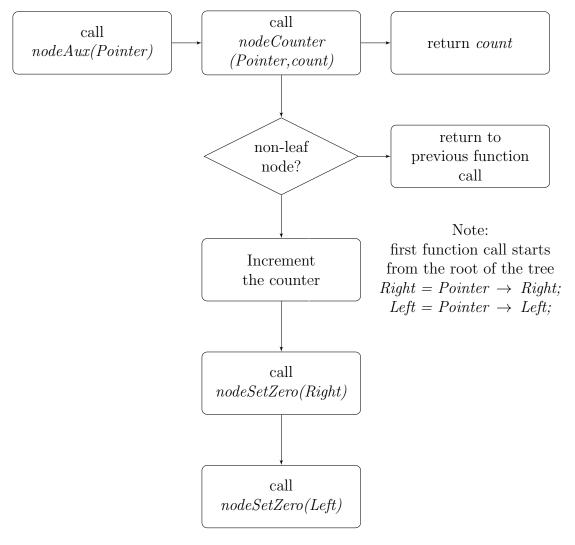


Figure 8: Flowchart for nodeCounter() and nodeCountAux()

## 5 nodeCounter and nodeCountAux

Those two functions are the last ones in the submitted code. The function node-Counter() takes the root of a binary tree as the argument. Inside the implementation, it will initialize a int variable count to 0. It will then call another recursive function nodeCountAux() and pass in the root of the binary tree and the count int variable as the arguments.

Inside the implementation of the func-

tion nodeCountAux(), the function will only be counting the non-leaf nodes. In order to determine whether the node is non-leaf node, it will extract the first character of the label on the node. If the extracted character is x then the node is a non-leaf node. Once the non-leaf node is detected, the function nodeCountAux() will increment the passed in variable. Since the counter is passed in by reference, the effect in each recursive call will have effect on the variable initialized in the function nodeCountAux().

The function nodeCountAux() returns the int variable count. (Refer to Figure 2)

## Part V

## Testing

To ensure at every stage of execution the program work correctly, the parts in the code that are labeled as uncomment to test are uncommented and extra outputs are produced other than normal evaluation outputs.

The following functions are called in order to producing the extra outputs:

```
seq_2D_char_input_test()
seq_2D_int_sort_test()
seq_2D_char_input_test()
```

A For loop is also used to test the matrix used for sorting routes using the weights.

The program is tested using the lab computer running Ubuntu. The computer is equiped with a i7 6700 CPU with 16GB of RAM. The maximum length of the binary string input is 30 characters. When I was testing 31-character input, I have to change the int arrays in the struct  $array_aux$  to long arrays. By doing so, the maximum length of the input strings is 31 characters long. Program uses about 4GB of the RAM while using the 30-bit binary string input with the arrays in the  $array_aux$  declared as int arrays. With a single 30-character long binary string input, the lab computer takes about 14 seconds to build the tree.

All of the following tests yield the correct results.

$\overline{x1}$	x2	Output	
0	0	0	•
0	1	1	
1	0	0	
1	1	1	
set 1	, test	ing 2 bits	•
testi	ng in	put array	before simplifying
testi	ng th	e 2D char	input array
1 1			
0 1			
after	$\sin_{\mathbf{l}}$	olification	
testi	ng th	e 2D int s	sort array
2 0 1	l 1 1	1	
100	0 0	2	
testi	ng th	e 2D char	input array
2 2			
2 1			
testi	ng th	e availabl	e route array
1 1			
00 tl	ne res	sult is: 0	
01 tl	ne res	sult is: 1	
10 tł	ne res	sult is: 0	
11 tl	ne res	sult is: 1	
trave	ersing	g the tree	
root	is: x	2	
1			
x2			
0			
_			

the number of non-leaf nodes is: 1

1 0 0 4				
$ \begin{array}{c cccc} x1 & x2 & \text{Output} \\ \hline 0 & 0 & 0 \end{array} $	$\frac{x1}{}$	x2	x3	Output
	0	0	0	0
$egin{array}{cccccccccccccccccccccccccccccccccccc$	0	0	1	0
1 1	0	1	0	0
2, testing 2 bits	0	1	1	1
ting input array before simplifying	1	0	0	0
ing the 2D char input array	1	0	1	1
l sine 2D chai input array	1	1	0	0
	1	1	1	0
<u>.</u>	$\overline{\text{set}}$	3, te	sting	3 bits
er simplification	test	ing i	nput	array be
ting the 2D int sort array	test	ting t	he 21	D char ii
1 1 2 1	1 0	1		
1112	0 1	1		
ting the 2D char input array	afte	er sin	plific	cation
)	test	ing t	he 2	D int sor
2	3 0	2 2 2	2 1	
1	1 1	1 1 2	2 2	
sting the available route array	2 1	1 1 2	2 2	
1	test	ing t	the 21	D char ii
3	1 0	1		
the result is: 0	0 1	1		
the result is: 1	test	ting t	he av	vailable 1
the result is: 1	3 5			
the result is: 1	5 5			
eversing the tree	000	the	resul	t is: 0
ot is: x2	001	the	resul	t is: 0
	010	the	resul	t is: 0
2	011	the	resul	t is: 1
	100	the	resul	t is: 0
	101	the	resul	t is: 1
	110	the	resul	t is: 0
e number of non-leaf nodes is: 2	111	the	resul	t is: 0
	tra	versir	ng the	e tree
	roo	t is:	х3	
	0			
	x2			
	1			
	x1			
	1			

x2
0
x3
0
the number of non-leaf nodes is: 4

$\overline{x1}$	x2	x3	Output
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0
	4 .		0.1.

set 4, testing 3 bits

testing input array before simplifying testing the 2D char input array

- 0 0 0
- 0 0 1
- 0 1 0
- 0 1 1
- 100
- 1 1 0
- 111

after simplification

testing the 2D int sort array

- 3 1 2 2 3 1
- $1\ 1\ 1\ 1\ 2\ 2$
- 201113

testing the 2D char input array

- 2 2 0
- 0 2 1
- 2 2 2
- 2 2 2
- 2 2 2
- 2 2 2
- 1 1 1

testing the available route array

- 0 1
- 1 3
- 7 6
- 000 the result is: 1
- 001 the result is: 1
- 010 the result is: 1
- 011 the result is: 1

100 the result is: 1
101 the result is: 0
110 the result is: 1
111 the result is: 1
traversing the tree
root is: x3
1
x2
0
x1
1
x3
1
the number of non-leaf nodes is: $3$

$\overline{x1}$	x2	x3	Output
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1
$\overline{\text{set}}$	5, tes	sting	3 bits

testing input array before simplifying testing the 2D char input array

- $0 \ 0 \ 0$ 001
- 0 1 0
- 0 1 1
- 100
- 101
- 1 1 0
- 1 1 1
- 000 the result is: 1
- 001 the result is: 1
- 010 the result is: 1
- 011 the result is: 1
- 100 the result is: 1
- 101 the result is: 1
- 110 the result is: 1
- 111 the result is: 1

traversing the tree root is: 1 1

the number of non-leaf nodes is: 0

$\overline{x1}$	x2	x3	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0
$\overline{\text{set}}$	6, tes	sting	3 bits

000 the result is: 0

001 the result is: 0

010 the result is: 0

011 the result is: 0

100 the result is: 0

101 the result is: 0

110 the result is: 0

111 the result is: 0

traversing the tree

root is: 0

0

the number of non-leaf nodes is: 0

$x_1$	x2	x3	x4	Output
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

set 7, testing 4 bits

testing input array before simplifying testing the 2D char input array

0 1 0 0

0001

1111

1 1 0 0

0 1 1 1

0 1 0 1

1 1 1 0

0 0 1 1

1011

after simplification

testing the 2D int sort array

3 2 2 2 4 1

4 2 2 2 4 1

 $2\ 0\ 2\ 2\ 2\ 2$ 

 $1\ 1\ 1\ 1\ 2\ 3$ 

testing the 2D char input array

 $2 \ 1 \ 0 \ 0$ 

 $0\ 2\ 0\ 1$ 

2 2 2 2

 $2\ 2\ 2\ 2$ 

```
2\ 2\ 2\ 2
                                                      1
2 2 2 2
                                                      x2
                                                      0
1\ 1\ 1\ 0
                                                      the number of non-leaf nodes is: 7
2 2 1 1
2 2 2 2
testing the available route array
3 2
4 4
1 5
14 7
0000 the result is: 0
0001 the result is: 1
0010 the result is: 0
0011 the result is: 1
0100 the result is: 1
0101 the result is: 1
0110 the result is: 0
0111 the result is: 1
1000 the result is: 0
1001 the result is: 0
1010 the result is: 0
1011 the result is: 1
1100 the result is: 1
1101 the result is: 0
1110 the result is: 1
1111 the result is: 1
traversing the tree
root is: x3
1
x4
1
x1
0
x2
0
x3
0
x1
1
```

x4

x1	x2	x3	x4	Output
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1 1	1 1	0	0 1	1 0
1	1	1	0	0
1	1	1	1	1
	3, test			
		_		before sin
	_		-	r input ar
0 1 (	_			P 333 31-1
0 0 0				
111				
11(	0 (			
0 1 3	l 1			
after	simp	olifica	ation	
testi	ng th	e 2D	int	sort array
2 1 2	2 2 3	1		
3 2 3	1 2 3	1		
4 1 2	2 2 3	1		
11(	11	2		
testi	ng th	ie 2D	cha	r input ar
2 1 (	0 (			
0 0 0	) 1			
2 2 2	2 2			
2 2 2	2 2			
2 1 1	l 1			
testi	ng th	ie ava	ailabl	le route ar
4 3				
7 3				

set 9, testing 30 bits		testing the available route array
testing input array before simplifying		0 30
testing the 2D char input array		evaluating 000000000000000000000000000000000000
- ,	1	ovariating occorronous control of the control of th
0 0 0 0 0 0 0 0 0	_	traversing the tree
after simplification		root is: x1
testing the 2D int sort array		0
1 1 0 1 1 1		x1
2 1 0 1 1 1		0
3 1 0 1 1 1		x2
4 1 0 1 1 1		0
5 1 0 1 1 1		x3
6 1 0 1 1 1		0
7 1 0 1 1 1		x4
8 1 0 1 1 1		0
9 1 0 1 1 1		x5
10 1 0 1 1 1		0
11 1 0 1 1 1		x6
12 1 0 1 1 1		0
13 1 0 1 1 1		x7
14 1 0 1 1 1		0
15 1 0 1 1 1		x8
16 1 0 1 1 1		0
17 1 0 1 1 1		x9
18 1 0 1 1 1		0
19 1 0 1 1 1		x10
20 1 0 1 1 1		0
21 1 0 1 1 1		x11
22 1 0 1 1 1		0
23 1 0 1 1 1		x12
24 1 0 1 1 1		0
25 1 0 1 1 1		x13
26 1 0 1 1 1		0
27 1 0 1 1 1		x14
28 1 0 1 1 1		0
29 1 0 1 1 1		x15
30 1 0 1 1 1		0
testing the 2D char input array		x16
$0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0\; 0$		0
$0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$		x17

0

x18

0

x19

0

x20

0

x21

0

x22

0

x23

0

x24

0

x25

0

x26

0

x27

0

x28

0

x29

0

x30

1

the number of non-leaf nodes is: 30

## Part VI

## Code

```
1
     /*
2
     * Run. cpp
3
         Created on: 15 April 2018
4
             Author: lw2016
5
6
     */
7
8
     #include < string >
     #include < vector >
9
     #include<iostream>
10
     #include<sstream>
11
12
     \#include < math.h >
13
14
     struct bdnode {
        std::string val;
15
        bdnode* left;
16
17
       bdnode* right;
18
     };
19
     typedef bdnode* bdt;
20
21
22
     struct array_aux {
23
24
     private:
25
26
        //number of digits in each input element
        int input_string_length;
27
28
29
        //length of array
        int array_length_1D;
30
31
32
        //length of the vector inpiut
        int vector_len;
33
34
35
        //int 1D array
36
        int *seq_1D_int;
```

```
37
38
       //n*2^n
39
       //2D char array
40
       char **seq_2D_char_input;
41
42
       //for accelerating the checking process
43
       bool *seq_1D_2D_available_space;
44
       //for sorting
45
       //bit 0: bit position
46
       //bit 1: bit = 0 counter
47
48
       //bit 2: bit = 1 counter
       //bit 3: max
49
       //bit 4: sum
50
       //bit 5: weight/priority
51
52
       int **seq_2D_int_sort;
53
54
     public:
55
       array_aux(int len, int len_of_vector) {
56
57
         input_string_length = len;
58
         array_length_1D = pow(2, input_string_length);
          vector_len = len_of_vector;
59
60
         //1D
61
62
         seq_1D_int = new int [array_length_1D];
63
         for (int i = 0; i < array_length_1D; i++) {
64
            seq_1D_int[i] = -1;
65
         }
66
67
         //2D & 1D
68
         seq_2D_char_input = new char*[vector_len];
69
70
         seq_1D_2D_available_space = new bool[vector_len];
71
72
         for (int i = 0; i < vector_len; i++) {
73
            seq_2D_char_input[i] = new char[input_string_length];
74
            seq_1D_2D_available_space[i] = false;
            for (int j = 0; j < input_string_length; <math>j++) {
75
              seq_2D_char_input[i][j] = '2';
76
```

```
}
77
          }
78
79
          //2D
80
81
           seq_2D_int_sort = new int*[input_string_length];
82
83
           for (int i = 0; i < input_string_length; i++) {
             seq_2D_{int\_sort[i]} = new int[6];
84
             seq_2D_int_sort[i][0] = i + 1;
85
             for (int j = 1; j < 6; j++) {
86
               seq_2D_int_sort[i][j] = 0;
87
88
             }
          }
89
        }
90
91
92
        ~array_aux() {
           if (seq_1D_int) {
93
             delete [] seq_1D_int;
94
95
           }
96
97
           if (seq_2D_char_input) {
             for (int i = 0; i < vector_len; i++) {
98
               delete [] seq_2D_char_input [i];
99
             }
100
             delete [] seq_2D_char_input;
101
          }
102
103
           if (seq_1D_2D_available_space) {
104
105
             delete [] seq_1D_2D_available_space;
           }
106
107
           if (seq_2D_int_sort) {
108
             for (int i = 0; i < input_string_length; i++) {
109
               delete [] seq_2D_int_sort[i];
110
             }
111
112
             delete [] seq_2D_int_sort;
113
           }
        }
114
115
116
        int return_array_length_1D() {
```

```
117
          return array_length_1D;
        }
118
119
        void seq_1D_int_set(int position, int val) {
120
          seq_1D_int[position] = val;
121
        }
122
123
        int seq_1D_int_return(int position) {
124
          return seq_1D_int[position];
125
        }
126
127
        void seq_1D_int_test() {
128
          for (int i = 0; i < vector_len; i++) {
129
             std::cout << i << '_' << seq_1D_int[i] << std::endl;
130
          }
131
132
          std::cout << std::endl;
133
        }
134
        void seq_1D_2D_char_input_set(int position, std::string binary_string) {
135
136
137
          if (seq_1D_int[position] = -1) {
            //find the location in the 2D char table
138
             //for inserting the new record
139
             bool found = false;
140
             int space_index = 0;
141
142
             while (!found && space_index < vector_len) {
143
               if (!seq_1D_2D_available_space[space_index]) {
144
                 found = true;
145
                 seq_1D_2D_available_space[space_index] = true;
146
147
               }
               else {
148
                 space_index++;
149
               }
150
            }
151
152
            //set the value on the mapping list
153
             seq_1D_int[position] = space_index;
154
155
156
            //set the values in the 2D input table
```

```
for (int i = 0; i < binary_string.length(); <math>i++) {
157
               seq_2D_char_input[space_index][i] = binary_string[i];
158
159
            }
160
          }
        }
161
162
163
        void seq_2D_char_input_set_bit(int position, int bit, char info) {
164
          int index_hashed = seq_1D_int[position];
165
166
          seq_2D_char_input[index_hashed][bit] = info;
167
        }
168
169
        char seq_2D_char_input_return(int x, int y) {
170
171
172
          int index_hashed = seq_1D_int[x];
173
          return seq_2D_char_input[index_hashed][y];
174
        }
175
176
177
        int seq_2D_char_input_weight(int position) {
178
          int index_hashed = seq_1D_int[position];
179
          int weight = 0;
180
181
          if (index_hashed != -1) {
182
             for (int i = 0; i < input_string_length; <math>i++) {
183
               if (seq_2D_char_input[index_hashed][i] != '0') {
184
                 weight += pow(2, input_string_length - 1 - i);
185
               }
186
187
             }
             return weight;
188
189
          return -1;
190
191
        }
192
193
194
        void seq_2D_char_input_test() {
195
          std::cout << "testing_the_2D_char_input_array" << std::endl;
196
```

```
197
           for (int i = 0; i < vector_len; i++) {
198
             for (int j = 0; j < input_string_length; j++) {
199
200
               std::cout << seq_2D_char_input[i][j] << '_';
             }
201
202
             std::cout << std::endl;
203
204
           std::cout << std::endl;
        }
205
206
207
         int check_available_routes() {
208
           int available_routes_count = 0;
209
           for (int i = 0; i < vector_len; i++) {
210
             if (seq_1D_2D_available_space[i]) {
211
212
               available_routes_count++;
213
             }
214
215
           return available_routes_count;
        }
216
217
218
        void delete_route(int position) {
219
220
           int index_hashed = seq_1D_int[position];
221
222
           for (int i = 0; i < input_string_length; <math>i++) {
             seq_2D_char_input[index_hashed][i] = '2';
223
224
           }
225
226
           seq_1D_int[position] = -1;
227
           seq_1D_2D_available_space[index_hashed] = false;
        }
228
229
230
         void seq_2D_int_sort_bit_set() {
231
232
           //fill in at position 1 and 2
233
           for (int i = 0; i < array_length_1D; i++) {
234
             if (seq_1D_int[i] != -1) {
235
               for (int j = 0; j < input_string_length; j++) {
                 if (\text{seq}_2D_\text{char}_\text{input}[\text{seq}_1D_\text{int}[i]][j] = '0') {
236
```

36

```
seq_2D_int_sort[j][1]++;
237
238
                 }
                 else if (seq_2D_char_input[seq_1D_int[i]][j] = '1') {
239
240
                   seq_2D_int_sort[j][2]++;
                 }
241
242
               }
243
            }
          }
244
245
          //fill in at position 3 and 4
246
247
          for (int i = 0; i < input_string_length; i++) {
248
            if (seq_2D_int_sort[i][1] \le seq_2D_int_sort[i][2]) 
               seq_2D_int_sort[i][3] = seq_2D_int_sort[i][2];
249
            }
250
            else {
251
252
               seq_2D_int_sort[i][3] = seq_2D_int_sort[i][1];
253
            seq_2D_int_sort[i][4] = seq_2D_int_sort[i][1]
254
              + seq_2D_int_sort[i][2];
255
          }
256
        }
257
258
        void seq_2D_int_sort_bit_sort(int bit_sort) {
259
260
          int swap_tmpp_0;
          int swap_tmpp_1;
261
262
          int swap_tmpp_2;
263
          int swap_tmpp_3;
264
          int swap_tmpp_4;
265
266
          //bubble sort the 2D int order array
267
          //using the sum
          for (int i = 0; i < input_string_length - 1; i++) {
268
            for (int j = 0; j < input_string_length - 1; <math>j++) {
269
270
               if (seq_2D_int_sort[j][bit_sort]
271
                 < seq_2D_int_sort[j + 1][bit_sort]) {
272
                 swap_tmpp_0 = seq_2D_int_sort[j + 1][0];
273
                 swap_tmpp_1 = seq_2D_int_sort[j + 1][1];
274
                 swap_tmpp_2 = seq_2D_int_sort[j + 1][2];
275
                 swap_tmpp_3 = seq_2D_int_sort[j + 1][3];
276
                 swap_tmpp_4 = seq_2D_int_sort[j + 1][4];
```

```
277
278
                 seq_2D_int_sort[j + 1][0] = seq_2D_int_sort[j][0];
279
                 seq_2D_int_sort[j + 1][1] = seq_2D_int_sort[j][1];
                 seq_2D_int_sort[j + 1][2] = seq_2D_int_sort[j][2];
280
                 seq_2D_int_sort[j + 1][3] = seq_2D_int_sort[j][3];
281
                 seq_2D_int_sort[j + 1][4] = seq_2D_int_sort[j][4];
282
283
284
                 seq_2D_int_sort[j][0] = swap_tmpp_0;
                 seq_2D_int_sort[j][1] = swap_tmpp_1;
285
                 seq_2D_int_sort[j][2] = swap_tmpp_2;
286
287
                 seq_2D_int_sort[j][3] = swap_tmpp_3;
288
                 seq_2D_int_sort[j][4] = swap_tmpp_4;
289
            }
290
          }
291
        }
292
293
        //sort: sum first, max after
294
295
        void seq_2D_int_sort_bit_run() {
          seq_2D_int_sort_bit_sort(4);
296
297
          seq_2D_int_sort_bit_sort(3);
        }
298
299
        int seq_2D_int_sort_return(int x, int y) {
300
          return seq_2D_int_sort[x][y];
301
        }
302
303
        void seq_2D_int_sort_test() {
304
305
          std::cout << "testing_the_2D_int_sort_array" << std::endl;
306
307
          for (int i = 0; i < input_string_length; i++) {
308
            for (int j = 0; j < 6; j++) {
309
               std::cout << seq_2D_int_sort[i][j] << '_';
310
            }
311
312
            std::cout << std::endl;
313
314
          std::cout << std::endl;
315
        }
316
```

```
void seq_2D_int_sort_priority_gen() {
317
318
           int priority_level = 1;
319
           seq_2D_int_sort [0][5] = priority_level;
320
321
322
           for (int i = 1; i < input_string_length; <math>i++) {
              if (seq_2D_int_sort[i][3] = seq_2D_int_sort[i-1][3]
323
                && \operatorname{seq}_2D_{\operatorname{int}}_{\operatorname{sort}}[i][4] = \operatorname{seq}_2D_{\operatorname{int}}_{\operatorname{sort}}[i-1][4])
324
                seq_2D_int_sort[i][5] = priority_level;
325
              }
326
              else {
327
328
                priority_level++;
                seq_2D_int_sort[i][5] = priority_level;
329
330
              }
           }
331
         }
332
333
       };
334
335
       bdt buildcompactbdt(const std::vector<std::string>& fvalues);
336
       std::string evalcompactbdt(bdt t, const std::string& input);
337
338
       int binaryToInt(const std::string num);
339
340
       std::string nodeNamer(const int level_label);
341
342
      void nodeList(bdt t);
343
344
       void nodeSetZero(bdt t);
345
346
347
      int nodeCounter(bdt t);
348
       void nodeCountAux(bdt t, int &count);
349
350
351
       bdt buildcompactbdt(const std::vector<std::string>& fvalues) {
352
353
         bdt bdt_array_root = new bdnode;
354
355
         if (fvalues.size() == 0)  {
           bdt_array_root ->val = "0";
356
```

```
bdt_array_root -> right = NULL;
357
358
           bdt_array_root -> left = NULL:
359
           return bdt_array_root;
        }
360
361
362
        //get the length of the input string
363
        int string_length = fvalues[0].length();
364
        int input_length = fvalues.size();
365
366
367
        array_aux *array_gen = new array_aux(string_length, input_length);
368
        //load values
369
370
        for (int i = 0; i < input_length; i++) {
371
           array_gen
372
          ->seg_1D_2D_char_input_set(binaryToInt(fvalues[i]), fvalues[i]);
373
        }
374
        //uncomment the following for testing
375
             std::cout << "testing input array before simplifying" << std::endl;
376
377
             array\_gen \rightarrow seq_2D\_char\_input\_test();
378
        if (array_gen -> check_available_routes()
379
        = array_gen->return_array_length_1D()) {
380
           bdt_array_root \rightarrow val = "1";
381
           bdt_array_root -> right = NULL;
382
           bdt_array_root -> left = NULL;
383
          return bdt_array_root;
384
        }
385
386
387
        //simplify the routes
        int count_before = 0:
388
        int count_after = 0;
389
        bool match_found = false;
390
391
        int distance = 0;
392
        int bit_position = 0;
393
        int length_1D = array_gen -> return_array_length_1D();
394
        int relative_position;
395
        int weight_i;
396
        int weight_relative;
```

```
397
        int difference;
398
        do {
399
           //check the termination condition, before simplification
400
           count_before = array_gen -> check_available_routes();
401
402
403
           for (int i = 0; i < length_1D; i++) {
             if (array\_gen \rightarrow seq_1D_int\_return(i) != -1) {
404
405
               match_found = false;
406
407
               bit_position = 0;
408
               while (bit_position < string_length && match_found == false) {
409
410
                 if (array_gen->seq_2D_char_input_return(i, bit_position) = '0')
411
412
413
                   distance = pow(2, (string\_length - bit\_position - 1));
414
                   relative_position = i + distance;
                   weight_i = array_gen->seq_2D_char_input_weight(i);
415
                   weight_relative = array_gen
416
417
                   ->seq_2D_char_input_weight (relative_position);
418
                   difference = weight_relative - weight_i;
419
420
                   if (array\_gen \rightarrow seq_1D\_int\_return(relative\_position) != -1
                     && difference == distance) {
421
422
                      array_gen -> delete_route (relative_position);
                      array_gen->seq_2D_char_input_set_bit(i, bit_position, '2');
423
                     match_found = true;
424
425
                   }
426
                 }
427
                 bit_position++;
428
            }
429
           }
430
431
432
           //check the termination condition, after simplification
433
           count_after = array_gen->check_available_routes();
434
435
        } while (count_before != count_after);
436
```

```
437
        array_gen->seq_2D_int_sort_bit_set();
438
        //sort: sum first, max after
439
        array_gen -> seq_2D_int_sort_bit_run();
440
441
442
        array_gen -> seq_2D_int_sort_priority_gen();
443
        //uncomment the following for testing
444
        //std::cout << "after simplification" << std::endl;
445
        //array_qen \rightarrow seq_2D_int_sort_test();
446
        //array\_gen \rightarrow seq_2D\_char\_input\_test();
447
448
        //create a table
449
450
        //record the available routes
        //calculate the corresponding weighting
451
452
        int **available_route = new int*[array_gen->check_available_routes()];
453
        for (int i = 0; i < array_gen->check_available_routes(); i++) {
454
           available_route[i] = new int[2];
455
           available_route[i][0] = 0;
456
457
           available\_route[i][1] = 0;
        }
458
459
        int weighting = 0;
460
        int available_route_pointer = 0;
461
462
        for (int i = 0; i < array_gen->return_array_length_1D(); i++) {
463
464
465
          if (array_gen->seq_1D_int_return(i) != -1) {
466
467
             available_route[available_route_pointer][0] = i;
             bit_position = 0;
468
             weighting = 0;
469
470
471
             while (bit_position < string_length) {</pre>
472
473
               if (array_gen->seq_2D_char_input_return(i,
                 array_gen->seq_2D_int_sort_return(bit_position, 0) - 1)
474
475
                 != '2') {
476
                 weighting += array_gen->seq_2D_int_sort_return(bit_position,
```

```
477
                    5);
               }
478
479
                bit_position++;
480
             }
             available_route[available_route_pointer][1] = weighting;
481
482
             available_route_pointer++;
483
           }
         }
484
485
         int swap_tmpp_0;
486
487
         int swap_tmpp_1;
488
489
         //bubble sort the route array
         //small first
490
         for (int i = 0; i < array_gen \rightarrow check_available_routes() - 1; <math>i++) {
491
492
           for (int j = 0; j < array_gen \rightarrow check_available_routes() - 1; <math>j++) {
493
             if (available\_route[j][1] > available\_route[j + 1][1]) {
               swap_tmpp_0 = available_route[j][0];
494
495
               swap_tmpp_1 = available_route[j][1];
496
497
                available\_route[j][0] = available\_route[j + 1][0];
498
                available\_route[j][1] = available\_route[j + 1][1];
499
                available_route [j + 1][0] = swap_tmpp_0;
500
                available\_route[j + 1][1] = swap\_tmpp\_1;
501
502
             }
503
           }
         }
504
505
506
         testing
             std::cout << "testing" the available route array" << std::endl;
507
             for (int \ i = 0; \ i < array\_gen \rightarrow check\_available\_routes(); \ i++)  {
508
               std::cout \ll available\_route[i][0] \ll ' ' \ll available\_route[i][1]
509
         //
                                                    \ll std :: endl;
510
             }
511
         512
             std::cout << std::endl;
513
         //start building the tree
514
         bdt bdt_tmpp = bdt_array_root;
515
516
         int level_label = 0;
```

```
517
         int no_ignore_node_count = string_length;
518
519
         //set the node counter in the router
         for (int i = 0; i < string_length; i++) {
520
521
           if (array_gen
522
           ->seq_2D_char_input_return(available_route[0][0], i)
523
             no_ignore_node_count --;
524
           }
525
        }
526
527
528
         //build the first route
         for (int i = 0; i < string_length; i++) {
529
           if (array_gen->seq_2D_char_input_return(available_route[0][0],
530
             array_gen \rightarrow seq_2D_int_sort_return(i, 0) - 1) = '1'
531
532
             level_label = array_gen->seq_2D_int_sort_return(i, 0);
533
             bdt_tmpp->val = nodeNamer(level_label);
534
             bdt_tmpp->right = new bdnode;
             bdt_tpp \rightarrow left = NULL;
535
             bdt_tmpp = bdt_tmpp->right;
536
537
             no_ignore_node_count --;
538
           else if (array_gen->seq_2D_char_input_return(available_route[0][0],
539
             array_gen \rightarrow seq_2D_int_sort_return(i, 0) - 1) = '0'
540
             level_label = array_gen->seq_2D_int_sort_return(i, 0);
541
542
             bdt_tmpp->val = nodeNamer(level_label);
             bdt_tmpp \rightarrow right = NULL;
543
             bdt_tmpp \rightarrow left = new bdnode;
544
545
             bdt_tmpp = bdt_tmpp -> left;
             no_ignore_node_count --;
546
           }
547
           if (no\_ignore\_node\_count == 0) {
548
             bdt_tpp \rightarrow val = "1";
549
             bdt_tmpp \rightarrow right = NULL;
550
551
             bdt_tpp \rightarrow left = NULL;
           }
552
         }
553
554
555
        //build the other routes
556
         //start from the root
```

```
//going along the existing route
557
558
        //compare the current node name with the node required in the route
559
        //if necessary, build extra nodes with the correct node names
        int route_pointer = 1; //skip the first route, which is built
560
        std::string current_node_string;
561
562
        int current_node_int;
563
        char retrieved_bit_value;
        std::string new_node_name;
564
        bool name_found;
565
        int name_position_counter;
566
567
568
        //array for storing the used nodes
        bool *used_node = new bool[string_length];
569
570
        while (route_pointer < array_gen -> check_available_routes()) {
571
572
573
          //reset the pointer
          //pointing the root
574
          bdt_tmpp = bdt_array_root;
575
576
577
          //reset the node counter in the route
578
          no_ignore_node_count = string_length;
579
580
          for (int i = 0; i < string_length; i++) {
            if (array_gen->seq_2D_char_input_return(
581
582
               available_route[route_pointer][0],
              i) = '2'
583
              no_ignore_node_count --;
584
            }
585
586
          }
587
          //reset the used node array
588
          for (int i = 0; i < string_length; i++) {
589
            used_node[i] = false;
590
591
          }
592
593
          //keep building the tree
          //until reaching the last element in the route
594
595
          while (no_ignore_node_count > 1) {
596
```

```
//get the current node/bit position/node name
597
598
             current_node_string = bdt_tmpp->val.substr(1,
599
               (bdt_tmpp \rightarrow val.length() - 1));
600
             std::stringstream ss1;
             ss1 << current_node_string;
601
             ss1 >> current_node_int;
602
603
            //set the current node
604
             //used state to true
605
             for (int i = 0; i < string_length; i++) {
606
607
               if (array_gen
608
              ->seq_2D_int_sort_return(i, 0) == current_node_int) {
609
                 used_node[i] = true;
               }
610
             }
611
612
             //retrieve the bit from the array
613
             //that stores the routing information
614
             retrieved_bit_value = array_gen->seq_2D_char_input_return(
615
               available_route[route_pointer][0], current_node_int - 1);
616
617
618
             //route the building sequence using the retrieved bit
             //add extra node if the pointer is pointing to a NULL position
619
             //skip adding new node if the node already exists
620
             if (retrieved_bit_value == '0') {
621
               if (bdt_tmpp \rightarrow left = NULL) {
622
623
624
                 //get the new node name
                 name_found = false;
625
626
                 name_position_counter = 0;
627
                 while (name_found == false) {
                   if (used_node[name_position_counter] = false && array_gen
628
                   ->seq_2D_char_input_return(available_route[route_pointer][0],
629
630
                   array_gen
631
                   ->seq_2D_int_sort_return(name_position_counter, 0) - 1)
632
                    != '2') {
633
634
                     new_node_name = nodeNamer(
635
                     array_gen
636
                     ->seq_2D_int_sort_return(name_position_counter, 0));
```

```
637
                      name\_found = true;
638
639
                    name_position_counter++;
640
                  }
641
                  //create the node
642
643
                  bdt_tmpp \rightarrow left = new bdnode;
644
                  //name the new node
645
                  bdt_tmpp->left->val = new_node_name;
646
647
648
                  //set the left and the right of the new node to NULL
                  bdt_tmpp->left->right = NULL;
649
                  bdt_tmpp \rightarrow left \rightarrow left = NULL;
650
               }
651
652
               //update the pointer
653
               bdt_tmpp = bdt_tmpp \rightarrow left;
               no_ignore_node_count --;
654
             }
655
             else if (retrieved_bit_value = '1') {
656
               if (bdt_tmpp->right == NULL) {
657
658
                  //get the new node name
659
                  name_found = false;
660
                  name_position_counter = 0;
661
662
                  while (name_found == false) {
                    if (used_node[name_position_counter] = false
663
664
                    && array_gen
665
                    ->seq_2D_char_input_return(available_route[route_pointer][0],
666
                    array_gen
667
                    ->seq_2D_int_sort_return(name_position_counter, 0) - 1) != '2')
668
                      new\_node\_name = nodeNamer
669
                      (array_gen -> seq_2D_int_sort_return (name_position_counter, 0))
670
671
                      name_found = true;
672
                    }
673
                    name_position_counter++;
674
                  }
675
676
                 //create the node
```

```
bdt_tmpp \rightarrow right = new bdnode;
677
678
679
                  //name the new node
                  bdt_tmpp->right->val = new_node_name;
680
681
682
                  //set the left and the right of the new node to NULL
683
                  bdt_tmpp \rightarrow right \rightarrow right = NULL;
                  bdt_tpp->right->left = NULL;
684
               }
685
               //update the pointer
686
               bdt_tmpp = bdt_tmpp->right;
687
688
               no_ignore_node_count --;
             }
689
             //retrieved bit = 2
690
             //route to the right
691
692
             //the\ same\ as\ retrieved\ bit = 1
693
             else {
694
                if (bdt_tmpp \rightarrow right = NULL) {
695
                  //get the new node name
696
697
                  //get the new node name
                  name_found = false;
698
                  name_position_counter = 0;
699
                  while (name_found == false) {
700
                    if (used_node[name_position_counter] = false &&
701
702
                    array_gen
703
                    ->seq_2D_char_input_return(available_route[route_pointer][0],
704
                    array_gen ->
                    seq_2D_{int\_sort\_return} (name_position_counter, 0) - 1)
705
                    != , 2, ) {
706
707
708
                      new\_node\_name = nodeNamer
709
                      (array_gen
710
                      ->seq_2D_int_sort_return(name_position_counter, 0));
711
                      name_found = true;
712
713
                    name_position_counter++;
                  }
714
715
                  //create the node
716
```

```
bdt_tmpp \rightarrow right = new bdnode;
717
718
719
                   //name the new node
                   bdt_tmpp->right->val = new_node_name;
720
721
722
                   //set the left and the right of the new node to NULL
723
                   bdt_tmpp->right->right = NULL;
724
                   bdt_tpp->right->left = NULL;
                }
725
                //update the pointer
726
727
                bdt_tmpp = bdt_tmpp->right;
728
              }
           }
729
730
           //fill in the value of the last element in the route
731
732
           current_node_string =
           bdt_tmpp \rightarrow val.substr(1, (bdt_tmpp \rightarrow val.length() - 1));
733
734
           std::stringstream ss;
735
           ss << current_node_string;
           ss >> current_node_int;
736
737
738
           retrieved_bit_value = array_gen->seq_2D_char_input_return(
              available_route[route_pointer][0], current_node_int - 1);
739
740
           if (retrieved_bit_value == '1') {
741
              bdt_tmpp \rightarrow right = new bdnode;
742
              bdt_tmpp->right->val = "1";
743
              bdt_tmpp->right->right = NULL;
744
              bdt_tmpp \rightarrow right \rightarrow left = NULL;
745
           }
746
747
           else {
              bdt_tmpp \rightarrow left = new bdnode;
748
              bdt_tpp \rightarrow left \rightarrow val = "1";
749
              bdt_tmpp->left->right = NULL;
750
              bdt_tpp \rightarrow left \rightarrow left = NULL;
751
           }
752
753
           route_pointer++;
         }
754
755
756
         nodeSetZero(bdt_array_root);
```

```
757
758
        delete array_gen;
759
        delete available_route;
760
761
762
        delete used_node;
763
        return bdt_array_root;
764
      }
765
766
767
      std::string evalcompactbdt(bdt t, const std::string& input) {
768
        bool found = false;
769
770
        std::string text;
        int number;
771
772
        if (t->val[0]!= 'x') {
773
          return t->val;
774
        }
775
776
777
        while (found == false) {
778
           text = t -> val;
779
           text = text.substr(1, (text.length() - 1));
780
781
           std::stringstream ss;
782
           ss << text;
783
           ss >> number;
784
           if (input[number - 1] = '1')
785
             t = t - > right;
786
             if (t->val == "1" || t->val == "0") {
787
               found = true;
788
             }
789
790
           }
           else if (input[number - 1] = '0') {
791
             t = t -> left;
792
             if (t->val = "1" | t->val = "0") {
793
794
               found = true;
795
             }
           }
796
```

```
}
797
798
         return t->val;
799
800
       int binaryToInt(const std::string num) {
801
802
803
         int decimal = 0;
         int length = num.length();
804
805
         for (int i = 0; i < length; i++) {
806
           if (\text{num}[\text{length} - \text{i} - 1] != '0')
807
              decimal = decimal + pow(2, i);
808
           }
809
810
         }
         return decimal;
811
812
       }
813
       std::string nodeNamer(const int level_label) {
814
815
816
         std::stringstream ss;
         ss << "x" << level_label;
817
         return ss.str();
818
      }
819
820
821
       void nodeList(bdt t) {
822
823
         if (t != NULL) {
           nodeList(t->right);
824
           std::cout << t->val << std::endl;
825
           nodeList(t->left);
826
         }
827
       }
828
829
       void nodeSetZero(bdt t) {
830
831
         if (t->val[0] = 'x')
832
           if (t->right == NULL \&\& t->left != NULL) {
833
              t \rightarrow right = new bdnode;
834
              t - > right - > val = "0";
835
              t \rightarrow right \rightarrow right = NULL;
836
```

```
837
                t \rightarrow right \rightarrow left = NULL;
838
             if (t->left == NULL && t->right != NULL) {
839
840
                t \rightarrow left = new bdnode;
                t \rightarrow left \rightarrow val = "0";
841
                t \rightarrow left \rightarrow right = NULL;
842
                t \rightarrow left \rightarrow left = NULL;
843
             }
844
             nodeSetZero(t->right);
845
             nodeSetZero(t->left);
846
          }
847
        }
848
849
       int nodeCounter(bdt t) {
850
          int count = 0;
851
          nodeCountAux(t, count);
852
853
          return count;
       }
854
855
        void nodeCountAux(bdt t, int &count) {
856
          \mathbf{if} (t->val[0] == 'x') {
857
             count++;
858
             nodeCountAux(t->right , count);
859
             nodeCountAux(t->left , count);
860
          }
861
        }
862
```

## References